### Towards Practical Coercion-Resistant Electronic Elections

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research & development

### Outline

- 1. Introduction
- 2. JCJ scheme a review
- 3. Our proposal
- 4. Client/Server trade-offs in universally verifiable elections
- 5. Conclusion

### **Electronic Voting Methods**

Supervised voting (off-line voting)



How to guarantee the "What You see is What You Vote For?

Remote Electronic Voting (on-line voting)



### Some desired properties of e-voting systems

#### n Eligibility: only legitimate voters can vote, and only once

- n Universal verifiability: All voters can verify that the final tally is correct
  - $\ensuremath{\,^{\ensuremath{\scriptstyle n}}}$  The votes they cast are included
  - n Only authorized votes are counted
  - $\ensuremath{\,{\scriptscriptstyle \sqcap}}$  No votes are changed during tallying

n Privacy: no adversary can learn any more about votes than is revealed by the final tally

- n Anonymity: hide map from voter to vote
- n Receipt-freeness: prohibit proof of vote
- n Coercion-freeness: adaptative

Stronger

Voters cannot prove whether or how they voted, even if they can interact with the adversary while voting.

### **Basic Tools**

#### n Building Blocks

 El Gamal cryptosystem (they need a variant of El Gamal in fact for their security proof)

n El Gamal cryptosystem: G a group of prime order *p*, *g* a generator of G

- n the secret key is x, the public key is  $h = g^x$ ,
- Encryption of *m* is  $c = (g^r, h^r m)$ ,
- n Decryption of *c* is  $(g^r)^{-x}(h^rm)$

# El Gamal cryptosystem

n Decryption (private key) can easily be distributed

n No need to trust a single entity

#### n Encryption is homomorphic

- n Multiplicative, or additive with a variant:  $E_h(m)^* E_h(m') = E_h(m^*m')$ 
  - $E_h(m)^* E_h(m') = E_h(m^*m')$
  - $E_h(m)^k = E_h(m^k)$
- n Computing on encrypted data is easy
- n Comparing the plaintexts of two ciphertexts (without decrypting them) is easy:

<sup>n</sup> Plaintext Equivalence Test (PET):  $PET(E_h(m1), E_h(m2) = 1 \text{ if } m1 = m2 \text{ and } 0 \text{ otherwise}$ 

#### n Re-encryption is easy : mix-nets can be efficiently implemented

- n For simulating an "anonymous channel"
- n For simulating "ballot shuffle"
- C = (g<sup>r</sup>, h<sup>r</sup>.m) can be transformed on a new ciphertext C' of *m* without knowing *m* and/or the secret key :  $C' = (g^{r+r'}, h^{r+r'}.m)$

### JCJ scheme\* – a review

#### n Basic ingredients:

- n Voter employs anonymous credential obtained during the registration phase
- n We don't know who voted (at time of voting) or what was voted
- n Valid credentials are required for vote to count
- n Voter can make "fake credentials" and vote multiple times
- n A coercer cannot tell whether a credential is correct or not
  - Attacker cannot tell whether a vote is valid or not

#### n Basic idea:

- n To mislead a coercer, the voter sends invalid ballot(s) as long as he is coerced, and a valid ballot as soon as he is not coerced
- n It suffices that the voter finds a window-time during which he is not coerced

#### \* Juels-Catalano-Jakobsson - WPES 2005

### Security model

n Registration:

- n Attacker cannot interfere with registration process
- n Before voting:
  - n Attacker can provide keying or other material to voter (even entire ballot)
- n During vote:
  - Votes may be posted anonymously (for strongest security) or semi-anonymously (for weaker guarantees)
  - n Bulletin board is universally accessible
- n At all times:
  - n Attacker has access to all public information, i.e., encrypted and decrypted ballots

#### Assumption. Voters trust their voting client.

# **Cast of Characters**



Receive their credential during the registration phase



Issue credentials in a distributed manner during the registration phase. They share an El Gamal secret key. R is the corresponding public key



Try to verify whether the coerced voter voted as prescribed



Manage the tallying process. They share an El Gamal secret key. T is the corresponding public key

### Registration

Ø The authorities generate a random value C<sub>s</sub>



Ø Mr Smith's credential is  $C_s$ . He can send a fake credential Fake $C_s$  to the coercer



### Voting

- $\emptyset$  Anatomy of a ballot: (E<sub>T</sub> (vote), E<sub>R</sub> (Credential), NIZKPs)
  - q Vote under coercion





q Revote



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#### Step 1: Check NIZKPs

Bulletin Board 1
E <sub>T</sub> (Bush), E <sub>R</sub> (FakeC <sub>B</sub> ), NIZKP
$E_{T}$ (Gore), $E_{R}$ ( $C_{D}$ ), NIZKP
E <sub>T</sub> (Bush), E <sub>R</sub> (FakeC <sub>S</sub> ), NIZKP
E <sub>T</sub> (Gore), E <sub>R</sub> (C <sub>S</sub> ), NIZKP
E <sub>T</sub> (Gore), E <sub>R</sub> (C <sub>J</sub> ), NIZKP
$-E_{T}$ (Bush), $E_{R}$ (C <sub>K</sub> ), NIZKP
E <sub>T</sub> (Bush), E <sub>R</sub> (C <sub>E</sub> ), NIZKP
•
E <sub>T</sub> (Bush), E <sub>R</sub> (C <sub>J</sub> ), NIZKP





#### Tallier 1 Tallier 2

#### **Ballots with invalid NIZKP are discarded**

#### **Step 2: Elimination of duplicates using PET**



#### Keep the last one for example

**Step 3: Mixing the ballots** 





Tallier 1





**Step 4: Mixing the list of valid credentials** 



Tallier 1





**Step 5: Checking credentials using PET** 





Authority 1

Authority 2



**Step 6: Decrypt valid votes** 





Authority 1

Authority 2



### Drawbacks

n quadratic overhead

- n *N* the number of voters, *V* the number of votes ( $V \ge N$ )
- n  $O(V^2)$  tests for duplicates
- n  $O(N^2)$  matching tests
- n denial of service attack

# Our proposal

#### n Building Blocks

- n ElGamal Cryptosystem (we also need in fact a variant of El Gamal for our security proof)
- n Mix Net
- n Zero-Knowledge proofs
- Credentials with a special structure: derived from "Boneh-Boyen-Sacham" or "Camenisch-Lysanskaya" signature schemes (Crypto'04)

# Designated verifier signature scheme

∨ Based on "Boneh-Boyen-Sacham's group signature scheme (Crypto 2004)

n Setup:

 $_{\mbox{\tiny D}}$  Generators g\_0, g, h of a cyclic group G of order p where DDH is hard

n Public key of the signer:  $PK = g_0^y$ ,

 $\square$  Secret key of the signer: SK = y

n "Signature" on a random message x:

n Choose a random value r

n Compute  $A = (g h^{x})^{1/(y+r)}$ 

n "Signature" on x = (A, r)

$$A^{y} = A^{-r}gh^{x}$$
 (1)  
 $A^{y+r}g^{-1}h^{-x} = 1$  (2)

n Designated Verification:

n Prove that  $Log_A(A^{-r}gh^x) = Log_{g_0}(PK)$  using a *Designated Verifier Proof* (Jakobsson-Sako – Impagliazzo)

n Only the (designated) verifier can be convinced by this proof

Deciding whether a pair (A, r) is a valid signature on a message x is equivalent to the DDH problem

# **Cast of Characters**



Receive their credential during the registration phase



Issue credentials in a distributed manner during the registration phase. They share a secret key of our DVS. R is the corresponding public key



Try to verify whether the coerced voter has voted as prescribed



Manage the tallying process. They share an El Gamal secret key. T is the corresponding public key

### Set-up

n Setup:

- n Registration authority:  $PK=g_0^y$ , SK=y
- $\ensuremath{\,^{\ensuremath{\mathsf{n}}}}$  Talliers: share y and an ElGamal secret key

n Registration

- n Credential: (A, r, x)
  - x and r are randomly chosen by R
  - A is computed as follows by R:  $A = (g h^x)^{1/(y+r)}$
- A credential is valid iff the voter knows two values x and r such that: A<sup>y+r</sup>=g h<sup>x</sup> (which is equivalent to A<sup>y+r</sup>g<sup>-1</sup>h<sup>-x</sup> = 1)
   Fake credential: (A, r, x')

# Registration

Ø The registration authorities generate in a distributed manner a DVS signature (A, r) on a random value x and prove to Mr Smith using a DVP that the signature is valid



 $\oslash$  Mr Smith's credential is (A, r, x). He can send a fake credential (A, r, x') to the coercer



### **Basic Facts about these credentials**

- n A passive coercer can't check if a credential is valid or not under the DDH assumption : given g, g<sup>a</sup>, g<sup>b</sup>, g<sup>c</sup> decide whether c = ab mod p or not.
- n **A coercer can't forge valid credentials under the q-SDH assumption** n q-SDH: given g,  $g^x$ , ...,  $g^{(x^q)}$ , find a pair (c, A) such that  $A^{x+c} = g$
- An active coercer can't check if a credential is valid or not (under the Strong DDH Inversion (SDDHI) assumption)

Strong DDH Inversion (SDDHI): Suppose that  $g \in \mathbb{G}$ is a random generator of order  $q \in \Theta(2^k)$ . Let  $\mathcal{O}_a(\cdot)$  be an oracle that, on input  $z \in \mathbb{Z}_q^*$ , outputs  $g^{1/(a+z)}$ . Then, for all probabilistic polynomial time adversaries  $\mathcal{A}^{(\cdot)}$  that do not query the oracle on x,

$$\Pr[a \leftarrow \mathbb{Z}_q^*; \ (x, \alpha) \leftarrow \mathcal{A}^{\mathcal{O}_a}(g, g^a); \ y_0 = g^{1/(a+x)}; \ y_1 \leftarrow \mathbb{G};$$
$$b \leftarrow \{0, 1\}; \ b' \leftarrow \mathcal{A}^{\mathcal{O}_a}(y_b, \alpha) : b = b'] < 1/2 + 1/\operatorname{poly}(k).$$

\* The SDDHI assumption holds in generic groups

### Anatomy of a ballot

Credential : A tuple (A, r, x) such that  $A^{y+r}g^{-1}h^{-x} = 1$ 

n Ballot

 $\cap (E_T[vote], E_T[A], E_T[A^r], E_T[h^x], F=m^x, P)$ 

□ *P* is a NIZKP of validity, that is :

- $E_T$  (vote) is an encryption of a valid vote
- Voter knows the plaintext related to  $E_T(A)$
- Voter knows the "discrete logarithm" of  $E_T[A^r]$  in the base  $E_T[A]$
- Voter knows the plaintext related to  $E_T[h^x]$  as well as the discrete logarithm x of this plaintext in the base h.
- Voter knows the discrete logarithm of F in the base m and that this discrete logarithm is equal to x

### Voting

#### $\oslash$ Anatomy of a ballot: (E<sub>T</sub> (vote), E<sub>T</sub> (A), E<sub>T</sub> (A<sup>r</sup>) E<sub>T</sub> (h<sup>x</sup>), m<sup>x</sup>, Proof)

q Vote under coercion





q Revote



 $E_{T}$  (Gore),  $E_{T}$  (A),  $E_{T}$  (A<sup>r</sup>)  $E_{T}$  (h<sup>x</sup>), m<sup>x</sup>, Proof2

Ith



# Tallying phase

#### **Step 1: Discard ballots with invalid proofs**

Bulletin Board 1
$E_{T}$ (Bush), $E_{T}$ (A), $E_{T}$ (A <sup>r</sup> ) $E_{T}$ (h <sup>x'</sup> ), m <sup>x'</sup> , Proof <sub>1</sub>
$E_{T}$ (Bush), $E_{T}$ (B), $E_{T}$ (B <sup>s</sup> ) $E_{T}$ (h <sup>y</sup> ), m <sup>y</sup> , Proof <sub>2</sub>
$E_{T}$ (Bush), $E_{T}$ (C), $E_{T}$ (C <sup>t</sup> ) $E_{T}$ (h <sup>z</sup> ), m <sup>z</sup> , Proof <sub>3</sub>
$E_{T}$ (Gore), $E_{T}$ (B), $E_{T}$ (B <sup>s</sup> ) $E_{T}$ (h <sup>y</sup> ), m <sup>y</sup> , Proof <sub>4</sub>
$E_{T}$ (Bush), $E_{T}$ (D), $E_{T}$ (D <sup>u</sup> ) $E_{T}$ (h <sup>v</sup> ), m <sup>w</sup> , Proof <sub>5</sub>
$E_{T}$ (Gore), $E_{T}$ (A), $E_{T}$ (A <sup>r</sup> ) $E_{T}$ (h <sup>x</sup> ), m <sup>x</sup> , Proof <sub>6</sub>





Tallier 1

Tallier 2

# Tallying phase

#### Step 2: Elimination of duplicates: the ballots that have the same fourth component







Tallier 2

#### Keep the last one for example



**Step 3: Mixing the ballots** 





Tallier 1

Tallier 2

Bulletin Board 3	-	Bulletin Board 4
$E_{T}$ (Bush), $E_{T}$ (A), $E_{T}$ (A <sup>r</sup> ) $E_{T}$ (h <sup>x'</sup> )		$E'_{T}$ (Gore), $E'_{T}$ (A), $E'_{T}$ (A <sup>r</sup> ), $E'_{T}$ (h <sup>x</sup> )
$E_{T}$ (Bush), $E_{T}$ (C), $E_{T}$ (C <sup>t</sup> ) $E_{T}$ (h <sup>z</sup> )		$E'_{T}$ (Gore), $E'_{T}$ (B), $E'_{T}$ (B <sup>s</sup> ), $E'_{T}$ (h <sup>y</sup> )
$E_{T}$ (Gore), $E_{T}$ (B), $E_{T}$ (B <sup>s</sup> ) $E_{T}$ (h <sup>y</sup> )		$E'_{T}$ (Bush), $E'_{T}$ (C), $E'_{T}$ (C <sup>t</sup> ), $E'_{T}$ (h <sup>z</sup> )
$E_{T}$ (Gore), $E_{T}$ (A), $E_{T}$ (A <sup>r</sup> ) $E_{T}$ (h <sup>x</sup> )		$E'_{T}$ (Bush), $E'_{T}$ (A), $E'_{T}$ (A <sup>r</sup> ), $E'_{T}$ (h <sup>x</sup> )

#### Reencrypt and permute each row

# Tallying phase

**Step 4: Checking credentials** 



Bulletin Board 4
$E'_{T}$ (Gore), $E'_{T}$ (A), $E'_{T}$ (A <sup>r</sup> ), $E'_{T}$ (h <sup>x</sup> )
$E'_{T}$ (Gore), $E'_{T}$ (B), $E'_{T}$ (B <sup>s</sup> ), $E'_{T}$ (h <sup>y</sup> )
$E'_{T}$ (Bush), $E'_{T}$ (C), $E'_{T}$ (C <sup>t</sup> ), $E'_{T}$ (h <sup>z</sup> )
$E'_{T}$ (Bush), $E'_{T}$ (A), $E'_{T}$ (A <sup>r</sup> ), $E'_{T}$ (h <sup>x'</sup> )

- 1. The authorities compute  $C=E'_{T}[A^{y+r}g^{-1}h^{-x}]$ from  $E'_{T}[A]$ ,  $E'_{T}[A^{r}]$ ,  $E'_{T}[h^{x}]$  and SK = y
- 2. Test whether C is an encryption of 1
  - 1. Power C to a fresh random number 'f' and jointly decrypt C<sup>f</sup>.
  - D[C<sup>f</sup>] =1 ? Yes = valid / No = invalid and discard ballots

# Tallying phase

**Step 5: Decrypt valid votes** 





Tallier 1

Tallier 2



### **Computational Definition of Coercion-Resistance (1)**



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### **Computational Definition of Coercion-Resistance (2)**



### **Computational Definition of Coercion-Resistance (3)**

DEFINITION 1. We define an election scheme ES as coercion resistant if for any polynomially-bounded adversary  $\mathcal{A}$ , any parameters n and  $n_{C}$ , and any probability distribution  $D_{n,n_{C}}$ , the quantity

 $\mathbf{Adv}_{\mathsf{ES},\mathcal{A}}^{c\text{-resist}} = \left| \mathbf{Succ}_{\mathsf{ES},\mathcal{A}}^{c\text{-resist}}(\cdot) - \mathbf{Succ}_{\mathsf{ES},\mathcal{A}}^{c\text{-resist-ideal}}(\cdot) \right|$ 

is negligible in all security parameters for any voter function Vo.

Intuitively, this definition means that in a real protocol execution, *A* learns nothing more than the election tally

Our protocol satisfies the coercion-resistant requirement (in the random oracle model) under the SDDHI assumption

### **Computational Definition of Verifiability**

Experiment 
$$\operatorname{Exp}_{\mathsf{ES},\mathcal{A}}^{ver}(k_1, k_2, k_3, n_C, n_V)$$
  
 $\{(sk_i, pk_i) \leftarrow \operatorname{register}(SK_{\mathcal{R}}, i, k_2)\}_{i=1}^{n_V};$  % voters are registered  
 $(\mathcal{BB}, X, P) \leftarrow \mathcal{A}(SK_{\mathcal{T}}, \{(sk_i, pk_i)\}_{i=1}^{n_V}, \text{"forge election"});$  %  $\mathcal{A} \text{ concocts full election}$   
 $(X', P') \leftarrow \operatorname{tally}(SK_{\mathcal{T}}, \mathcal{BB}, n_C, \{pk_i\}_{i=1}^{n_V}, k_3);$  % tally is taken on  $\mathcal{BB}$   
if  $X \neq X'$  % does  $\mathcal{A}$ 's tally differ from correct  $\mathcal{BB}$  tally?  
and verify $(PK_{\mathcal{T}}, \mathcal{BB}, n_C, X, P) = \text{`1' then}$  % does function verify accept?  
else

output '0';

 $\mathbf{Succ}^{\mathit{E}}_{\mathsf{ES},\mathcal{A}}(\cdot) = \mathsf{Pr}[\mathbf{Exp}^{\mathit{E}}_{\mathsf{ES},\mathcal{A}}(\cdot) = `1'] \text{ should be negligible}$ 

Our protocol satisfies the verifiability requirement (in the random oracle model) under the q-SDH assumption

# Client/Server trade-offs in universally verifiable elections

n Setup:

- Generators g<sub>0</sub>,g,h,m of a cyclic group G of order p (DDH problem is hard)
- $\square$  Registration authority: PK=g<sub>0</sub><sup>y</sup>,SK= y
- n Talliers: share y and an ElGamal secret key
- n Encoding of votes for L candidates:
  - n *M*: Upper bound on number of voters.
  - n candidate  $1 \rightarrow 1$ , candidate  $2 \rightarrow M$ , ..., candidate  $L \rightarrow M^{L-1}$ .

### Generation of the ballots

n Ballot for the candidate j: (A, r, x)

- Where  $x = M^{j}$  and r is randomly chosen by R
- A is computed as follows by R:  $A = (g h^x)^{1/(y+r)}$

n The ballot is valid *iff* :  $A^{y+r} = g h^x$  (which is equivalent to  $A^{y+r}g^{-1}h^{-x} = 1$ )

### Voting

n A vote for candidate j: (E[A], E[A<sup>r</sup>], E[h<sup>x</sup>], P) where  $x = M^{j}$ 

□ *P* is a NIZKP of validity, that is :

- *E(vote)* is an encryption of a valid vote
- Voter knows the plaintext related to E(A)
- Voter knows the "discrete logarithm" of *E*[*A*<sup>*r*</sup>] in the base *E*[*A*]
- Voter knows the plaintext related to *E*[*h*<sup>x</sup>] as well as the discrete logarithm *x* of this plaintext in the base *h*.

#### **Step 1: Discard ballots with invalid proofs**

<b>Bulletin Board 1</b>
$E_{T}(A), E_{T}(A^{r}) E_{T}(h^{x}), Proof_{1}$
$E_{T}(A), E_{T}(A^{r}) E_{T}(h^{x}), Proof_{2}$
$E_{T}(C), E_{T}(C^{t}) E_{T}(h^{z}), Proof_{3}$

 $E_{T}(D), E_{T}(D^{u}) E_{T}(h^{w}), Proof_{4}$ 





Tallier 1

Tallier 2

**Step 4: Checking valid ballots** 



Bulletin Board 2
$\begin{aligned} & E_{T}(A), E_{T}(A^{r}) E_{T}(h^{x}) \\ & E_{T}(A), E_{T}(A^{r}) E_{T}(h^{x}) \end{aligned}$
$\frac{E_{T}(C), E_{T}(C^{t}) E_{T}(h^{z})}{E_{T}(h^{z})}$
$E_{T}(D), E_{T}(D^{u}) E_{T}(h^{w})$

- 1. The authorities compute  $C=E[A^{y+r}g^{-1}h^{-x}]$  from  $E[A], E[A^r], E[h^x]$  and SK = y
- 2. Test whether C is an encryption of 1
  - 1. Power C to a fresh random number 'f' and jointly decrypt C<sup>f</sup>.
  - D[C<sup>f</sup>] =1 ? Yes = valid / No = invalid and discard ballots



Step 6: compute the result using the homomorphism

	Bulletin Board 3
$E_{T}(h^{x})$	)
$E_{T}(h^{x})$	)
E <sub>T</sub> (h <sup>w</sup>	)



Tallier 1Tallier 2

### Conclusion

- n The JCJ scheme is promising, but not efficient
- N We design a practical (with linear work factor), publicly verifiable and coercion- resistant voting scheme (with respect to JCJ's model) for remote elections
- n Not just practical, but essential for Internet voting!
- n Open problem: how to remove the assumption related to the voter's computer?